

Getting higher resolution rainfall estimates: X-band radar technology and multifractal drop distribution

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Abstract Hydrologists have been waiting for some time to have radar data with a resolution higher than the kilometre scale, especially for urban applications. This is now achievable with the help of polarimetric X-band radars, not only because of their higher frequency, but also because they are much more affordable and versatile. X-band radar networks are thus planned around megalopolises. However, to fully take advantage of the sophisticated polarimetric “self-calibration” requires further investigations of fundamental questions. For instance, ad-hoc homogeneity approximations and/or factorization of the drop distribution have led to the common practice to average several scans, and therefore to degrade the measurement resolution in an attempt to reduce the coherent backscattering due to heterogeneity of the drop distribution. With the help of high-resolution data from an infrared optical spectro-pluviometer, we come back to the question of the insights brought by multifractals on the corresponding statistical bias.

Key words X-band radar; urban hydrology; drop distribution; multifractals

INTRODUCTION

Weather radars remain the only measuring devices that provide space-time estimates of rainfall. However, their classical resolution scale of one kilometre does not meet the relevant scales of urban hydrology (e.g. Berne *et al.*, 2004), especially when there are increasing concerns about the sustainable development of large cities in the context of climate change. On the one hand, urbanisation sprawl induces considerable change in landscape and land-use, and requires more detailed observations and integrated predictions of the water balance. On the other hand, IPCC foresees an increase of hydrological extremes and heat waves (Solomon *et al.*, 2007). A key driver is the extreme variability of the rain field from planetary scales (Lovejoy *et al.*, 2008) to centimetre scales (Lilley *et al.*, 2006): the rain-rate, which is the classical precipitation observable, is strongly scale dependent. This feature corresponds to the fact that rain accumulation is a (mathematical) singular measure (Schertzer *et al.*, 2010). The latter property has many important and practical consequences, especially for small-scale observations. These consequences become even more important due to the recent breakthrough of dual polarimetry (Testud *et al.*, 2000; Le Bouar *et al.*, 2001; Anagnostou *et al.*, 2004; Maki *et al.*, 2005; Matrosov *et al.*, 2005; Berne & Uijlenhoet, 2006). Dual polarimetry provides a radar self-calibration with the help of an estimate of the drop size distribution (DSD) based on the differential reflectivity varying in response to drops flattening with their size. This self-calibration is rather indispensable to removing the rain attenuation bias, which increases with the radar frequency and which has been a long lasting problem (Hitschfeld & Bordan, 1954). X-band radars became thus able to measure, and not only to detect, rainfall. They have several attractive features: reduced transmitted power and antenna size, and reduced sensitivity to ground clutter. These features are particularly attractive in rugged topography and urban areas. For instance, X-net (Maki *et al.*, 2008) is a network around Tokyo of three polarimetric X-band radars run by INED and 4 X-band radars managed by other institutions. Similar networks are to be deployed around 10 other major towns in Japan (Maki *et al.*, 2010) and four polarimetric X-band radars of CASA-IP1 are deployed around Oklahoma City (Chandrasekar *et al.*, 2007). There are several other projects, including one in the Paris, France, region led by the Chair “Hydrology for Resilient Cities” of Ecole des Ponts ParisTech, sponsored by VEOLIA. All these projects are focused on getting higher resolution rainfall estimates and call attention on needed improvements of the present retrieval schemes of rain-rate from reflectivity and to consider

more realistic assumptions than those that are currently used.

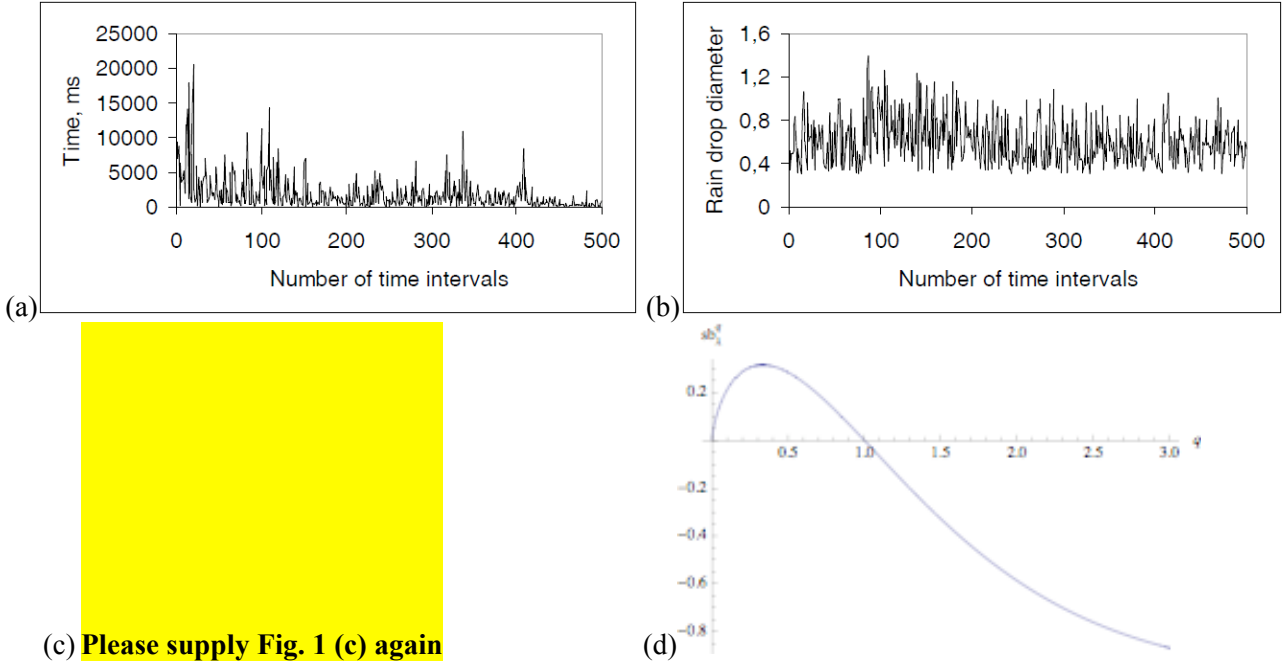


Fig. 1 (a)–(d): Example of an infrared OSP record: (a) series of the time interval durations between drops, (b) corresponding series of the drop diameters, (c) log-log plot of the statistical moments (of orders $q = 0.5, 1.5, 2.5, 3.5$, from bottom to top) of the drop volumes vs. the time resolution $\lambda (= \Delta t/T$, Δt being the observation time scale, T the sample length (= 1 day). The latter exhibit two clear scaling ranges (power laws, i.e. straight lines): from 1 ms to about 2 s and from 7 min up to 1 day (total sample time). Both ranges, separated by a transition plateau, yield the same universal parameter estimates (equation (18)). The latter are used to obtain (d) a semi-analytical estimate (equation (16)) of the corresponding relative speckle bias $sb_{\lambda}^{(q)}$ vs the statistical moment orders q 's. One may note that this bias is already of 33% and 60% for $q = 1.5, 2$, respectively.

RADAR REFLECTIVITY AND RAINFALL VARIABILITY

Developments of polarimetric retrieval schemes have been mostly focused on getting observables less dependent on the DSD variability, and are based on the differential reflectivity Z_{DR} or the specific differential phase shift K_{DP} in order to get robust correction schemes for the absorption. The basic problem of the “speckle” effect or “drop rearrangement” has remained rather untouched. This effect results from the ubiquitous hypothesis of a homogeneous distribution of the drops that is mathematically convenient to factorize the drop distribution into a Poisson distribution of centres and a translation invariant DSD, although it is physically implausible. This hypothesis was rather compulsory when the computation means were rather limited and radars were mainly used for rain detection (Lawson & Uhlenbeck, 1950), whereas the turbulent wind as well as the coalescence processes tend to cluster drops, rather than to homogeneously distribute them in space and time. This clustering was empirically checked by (Lovejoy & Schertzer, 1990): the fractal distribution of raindrops on horizontal sections exhibits a fractal dimension of order 1.8, instead of 2 for a homogeneous distribution. This result was disputed by (Gabella *et al.*, 2001; Uijlenhoet *et al.*, 2009) and refined by (Desaulnier-Soucy *et al.*, 2001; Lilley *et al.*, 2006) with the help of 3D multifractal analyses of rain field volumes of about a cubic metre. The impact of the speckle effect was tentatively estimated by (Lovejoy *et al.*, 1996) with the help of a comparison of the effective reflectivity field Z_e , which is the Fourier transform of the backscatter distribution:

$$Z_{e,\lambda} = \left| \int_{B_{\lambda}} \sigma_{\lambda}(x) e^{ik_r \cdot x} dx \right|^2 \quad (1)$$

and the traditional radar reflectivity factor Z_λ :

$$Z_\lambda = \int_{B_\lambda} \sigma_\lambda(x)^2 dx \quad (2)$$

Both reflectivities are written with an apparent continuous integration and non-dimensional variables for convenience: $\lambda = L/l$ denotes the non-dimensional radar resolution, i.e. the scale ratio of a given outer scale L with respect to the radar pulse length l ($l \approx 100m$ for X-band radars) and B_λ is the corresponding radar pulse volume; the resolution $\Lambda = L/\eta$ corresponds to the inner scale η of the backscatters/rain field variability and the integration is in fact discrete; $\sigma_\lambda(x) = v(x)/\text{vol}(B_\lambda)$ is the relative volume of the drop volume $v(x)$ centred at the non-dimensional location $x = r/L$; k_r is the non-dimensional radar pulse wave-number (i.e. also adimensionalized by L). The classical model corresponds to incoherent small-scale variability (homogeneous distribution of backscatters): the backscatter phases in equation (2) are independent identically distributed random variables and therefore for a large number of drops, the cross terms cancel leading to $Z_{e,\lambda} \approx Z_\lambda$. The deviation from unity of the ratio of the corresponding statistical moment of order q ($\langle \cdot \rangle$ denotes the ensemble average):

$$sb_\lambda^{(q)} = \langle Z_{e,\lambda}^q \rangle / \langle Z_\lambda^q \rangle - 1 \quad (3)$$

measures the speckle bias of order q . To take into account the attenuation, one has to distinguish the apparent (attenuated) radar reflectivity Z_a from the equivalent radar reflectivity Z_e through:

$$Z_a(x) = Z_e(x) \exp\left(-\int_0^x A(s) ds\right) \quad (4)$$

where A is the attenuation rate by unit distance. The classical DSD is defined by an exponential probability distribution (Marshall & Palmer, 1948):

$$N(D) = N_0 \exp(-D/D_m) \quad \text{should this be equation (5)?}$$

where N is the concentration by drop size diameter D , N_0 is the “intercept parameter”, and D_m is the “median volume” diameter. The mathematical convenience of an exponential distribution is that its statistical moments of order q are all proportional to the corresponding power of the median volume diameter, as for a deterministic distribution:

$$\langle D^q \rangle = \int dD D^q N(D) = N_0 D_m^q \Gamma(m+1). \quad (5)$$

This convenient property may easily turn out to be a weak point, because it corresponds to a case of weak variability. Nevertheless, its practical success is that it easily yields a Z - R relationship for incoherent scattering, because Z corresponds to $(\sigma \propto D^3)$:

$$Z \propto \int dD D^6 N(D) \quad (6)$$

whereas the rain-rate R corresponds to:

$$R \propto \int dD D^3 v(D) N(D) \quad (7)$$

where the terminal velocity is assumed to be scaling:

$$v(D) \propto D^\delta \quad (8)$$

The precise value of the exponent δ is a source of uncertainty: $\delta = 2$ corresponds to the Stokes law (in laminar flows), whereas $\delta = 1/2$ is commonly used in the high Reynolds number regime and according to Atlas & Ulbrich (1977) $\delta = 2/3$ is more accurate. This yields two types of Z - R relationships:

$$Z = aR^b; \quad b = 6/(3 + \delta), \quad a = N_0^{1-b} \Gamma(7) / \Gamma(1 + 6/b)^b \quad (9)$$

$$Z = AR; \quad A = D_m^{6(b-1)/b} \Gamma(7) / \Gamma(1 + 6/b) \quad (10)$$

that are parameterized by N_0 and D_m , respectively. Unfortunately, none of them is able to predict R accurately from Z , because both N_0 and D_m have large random fluctuations. In contrast, due to the relation existing between Z_{DR} and D_m , the above Z - R relation parameterized by D_m (equation (10)) can be transformed into a two-parameter estimator $R(Z, Z_{DR})$, which can be less dependent on the DSD variability (Chandrasekar *et al.*, 1990; Gorgucci *et al.*, 1991). As suggested by (Jameson, 1991) and developed by Gorgucci & Scarchilli (1997) K_{DP} can be used as well. A relatively less dependence on the DSD is obtained with the help of slightly more general definitions (Testud *et al.*, 2001) of the parameters N_0 and D_m (then called “mean volume diameter”):

$$D_m = 4 \langle D^4 \rangle / \langle D^3 \rangle; \quad N_0 = 4^4 LWC / (\pi \rho D_m^4) \quad (11)$$

that are defined for any DSD. However, they are only two among many possible parameters of a probability distribution and whose physical significance largely depends on the type of the actual DSD. Furthermore, their relative generality does not preclude any speckle effect. The latter has been indirectly estimated by (Le Bouar *et al.*, 2001) for Z_a in the framework of the ZPHI algorithm (Testud *et al.*, 2000) as of the order of $\pm 25\%$ for $\delta A/A$ and $\pm 20\%$ for $\delta R/R$ for 10 “independent” samples of the same segment. These estimates were used to estimate the standard error of Φ_{DP} , which was obtained by averaging over enough adjacent gates (11 in the precise case). This also corresponds to the general practice of averaging on a large enough number of samples to smooth out the phase dependency of Z_e and therefore to become closer to incoherent scattering.

SPECKLE AND MULTIFRACTALITY

Due to the scale dependence of various expressions derived above, the respective importance of various terms could be theoretically deduced from their scaling behaviour. This is particularly simple for the radar reflectivity Z_λ and the rain-rate R_λ because both correspond to an integration of a given power of the relative drop volume σ or of the corresponding diameter D . There are general reasons to believe that both are multifractal as the rain-rate is (Schertzer & Lovejoy, 1987; Lovejoy & Schertzer, 1995). This was analysed by Tchiguirinskaia *et al.* (2003) on a high-resolution time series of diameters and time intervals obtained by Salles *et al.* (1998) with the help of an infrared optical spectro-pluviometer (OSP), see Fig. 1(a)–(c). In particular, the statistical moments exhibit multiscaling/multifractality, i.e. for any (positive) order q :

$$\langle \sigma_\lambda^q \rangle = \lambda^{K_\sigma(q)} \langle \sigma_1^q \rangle \quad (12)$$

where the moment scaling function $K_\sigma(q)$, with $K_\sigma(0)=0$ due to the probability normalisation, is convex and nonlinear, except for the exceptional case of mono-/uni-fractality. With the help of the technique of “normalized power densities” (Schertzer *et al.*, 1997, 2002; Schertzer & Lovejoy, 2011) that yields:

$$\left\langle \left(\int_{B_\lambda} \sigma_\lambda^p dx \right)^q \right\rangle \propto \lambda^{K_\sigma(q,p)}; \quad K_\sigma(q,p) \equiv K_\sigma(q,p) - qK_\sigma(p) \quad (13)$$

one obtains from equations (2), (6)–(7), (12)–(13):

$$\langle Z_\lambda^q \rangle \propto \lambda^{K_Z(q)}; \quad K_Z(q) = K_\sigma(q,2); \quad \langle R_\lambda^q \rangle \propto \lambda^{K_R(q)}; \quad K_R(q) = K_\sigma(q,2/b). \quad (14)$$

On the other hand, due to the fact that the effective reflectivity field Z_e (equation (1)) is the energy spectrum component for the radar wave number k_r and is therefore the Fourier transform of the autocorrelation function of the backscatter field σ_λ , one obtains, still with the help of the technique of “normalized power densities”:

$$\langle Z_{e,\lambda}^q \rangle \propto k^{K_Z(q)}; \quad K_Z(q) = K_\sigma(q,2). \quad (15)$$

As a consequence, the speckle bias scales like:

$$sb_{\lambda}^{(q)} = (k/\lambda)^{K_z(q)} - 1 \quad (16)$$

To obtain an estimate of this bias, let us consider that the backscatter field σ_{λ} is a universal multifractal field (Schertzer & Lovejoy, 1997) for which:

$$K(q, p) \equiv K(q, p) - qK(p) = \frac{C_1}{\alpha - 1} p^{\alpha} (q^{\alpha} - q); \quad K(q) \equiv K(q, 1) \quad (17)$$

where the co-dimension of the mean field $C_1 \geq 0$ measures the mean intermittency ($C_1 = 0$ implies that the field is homogeneous) and the multifractality index $0 \leq \alpha \leq 2$ measures how the intermittency varies with various thresholds or statistical moment order ($\alpha = 0$ corresponds to a mono-/uni-fractal, whose intermittency of the extremes and the mean are the same). The following estimates (Tchiguirinskaia *et al.*, 2003) obtained for the multifractal distribution of raindrop volumes in time:

$$C_{1,\sigma} \approx 0.35; \quad \alpha_{\sigma} \approx 0.82 \quad (18)$$

yield the speckle bias $sb_{\lambda}^{(q)}$ displayed in Fig. 1(d), for $k_r/\lambda \approx 3$ by considering the wavelength of an X-band radar $k_r^{-1} \approx 3 \text{ cm}$ and the inter-drop distance $\lambda^{-1} \approx 1 \text{ cm}$. Using the Marshall-Palmer relationship (equation (9)), one can crudely estimate similar relative biases for the rain-rate, but for respective moment orders $q_R = qb$, with e.g. $b = 18/11 \approx 1.6$. It is worthwhile to note that a more detailed analysis is required for moment orders q_R 's larger than the critical order q_D of the rain-rate statistical moment divergence, which is presumably of the order of 3 (Lovejoy & Schertzer, 1995). Therefore, more detailed analyses are required to assess the biases of higher order.

DISCUSSION AND CONCLUSION

Higher resolution rainfall data are now available with the help of polarimetric X-band radars. However, in spite of the increasing sophistication of the retrieval algorithms, the speckle problem remains rather unsolved. Using a multifractal approach, we first obtain a general expression of the speckle bias (equation (16)). Using empirical estimates of the multifractal parameters of the drop volume distribution obtained by Tchiguirinskaia *et al.* (2003) on a high-resolution OSP time series (Salles *et al.*, 1998), we obtain speckle bias estimates already of 33% and 60% for the statistical moments of order $q = 1.5; 2$, respectively, contrary to a previous, global estimate of 25% (Le Bouar *et al.*, 2001). We also point out similar biases for the rain-rate, as well as the necessity to pursue more detailed analyses. Nevertheless, the present results show that there is a necessity to work directly on the effective reflectivity rather than on the standard reflectivity.

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